§3.2 4d $\mathcal{N}=1$ SCFTs and Seiberg duality

Consider $\mathcal{N}=1$ supersymmetric QCD with

- gauge group $SU(N_c)$
- N_f flavors of quarks, Q^i in N_c rep. and $\tilde{Q}_{\tilde{i}}$ in the $\overline{N_c}$ rep. $(i, \tilde{i} = 1, \ldots, N_f)$

→ anomaly free global symmetry:

	$SU(N_f)$	$\times\ SU(N_f)$	$\times\ U(1)_B$	$\times\ U(1)_R$
Q	N_f	1	1	$\frac{N_f - N_c}{N_f}$
$\tilde{Q}$	1	$\overline{N_f}$	-1	$\frac{N_f - N_c}{N_f}$

table 1

gauge invariant operators:

$$M^i_{\tilde{i}} = Q^i \tilde{Q}_{\tilde{i}}$$

$$B^{[i_1, \ldots, i_{N_c}]} = Q^{i_1} \cdots Q^{i_{N_c}} \quad \text{for } N_f \geq N_c$$

$$\tilde{B}_{[\tilde{i}_1, \ldots, \tilde{i}_{N_c}]} = \tilde{Q}_{\tilde{i}_1} \cdots \tilde{Q}_{\tilde{i}_{N_c}} \quad \text{for } N_f \geq N_c$$

can show: for $N_f \geq N_c$ quantum theory has moduli space of vacua $\mathcal{M}$

<u>but</u> for $N_f < N_c$ there is <u>no</u> ground state

origin of moduli space $M = B = \tilde{B} = 0$ enjoys unbroken $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$ symmetry

Beta function is given by

$$\beta(g) = -\frac{g^3}{16\pi^3} \frac{3N_c - N_f + N_f \gamma(g^2)}{1 - N_c \frac{g^2}{8\pi^2}}$$

$$\gamma(g^2) = -\frac{g^2}{8\pi^2} \frac{N_c^2 - 1}{N_c} + \mathcal{O}(g^4)$$

$\rightarrow$ for $N_f > 3N_c$ theory is not asymptotically free

Will argue that for $3N_c/2 < N_f < 3N_c$

$SU(N_c)$ with N_f flavors at origin of $\mathcal{M}$ "electric phase" — Theory A

flows

dual

SCFT

flows

$SU(N_f - N_c)$ with N_f flavors and gauge singlet M "magnetic phase" — Theory B

Quarks and gluons of Theory A/B can be thought of as magnetic monopoles of Theory B/A

$$3(N_f - N_c) - N_f < 0$$

$$\Leftrightarrow \quad 2N_f - 3N_c < 0 \quad \Leftrightarrow \quad N_f < \tfrac{3}{2} N_c$$

$\rightarrow$ theory B is free in the IR for $N_f < \frac{3}{2} N_c$
while theory A is strongly coupled

$\rightarrow$ "weak coupling - strong coupling" duality

The two theories should have the same global symmetries $\rightarrow$ symmetries of SCFT fixed point

So where is this fixed point?

Take $N_c, N_f \rightarrow \infty$ and $N_c g^2$, $\frac{N_f}{N_c} = 3 - \varepsilon$ fixed

$$\rightarrow \beta(g) = \frac{g}{16\pi^2} g^2 N_c \, \frac{3 - N_f/N_c + N_f/N_c \, \gamma(g^2)}{1 - N_c \frac{g^2}{8\pi^2}}$$

$$\gamma(g^2) \rightarrow - \frac{g^2 N_c}{8\pi^2}$$

Then $\beta(g) = 0 \Leftrightarrow 3 - (3-\varepsilon) + (3-\varepsilon)\left(-\frac{g^2 N_c}{8\pi^2}\right) = 0$

$$\Leftrightarrow 8\pi^2 \varepsilon = (3-\varepsilon) g^2 N_c$$

$$\Leftrightarrow N_c g_*^2 = \frac{8\pi^2}{3} \varepsilon + \mathcal{O}(\varepsilon^2)$$

$\rightarrow$ non-trivial fixed-point

claim: for every $3N_c/2 < N_f < 3N_c$

Superconformal algebra for 4d $\mathcal{N}=1$ implies:

$$\Delta \geq \frac{3}{2}|R|$$

(see §2.2 and tables therein)

inequality is saturated for chiral operators $\Delta = \frac{3}{2}R$

and anti-chiral ones $\Delta = -\frac{3}{2}R$

Operator product gives: $\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_i C^i_{12}(x)\mathcal{O}_i(0)$

with $R(\mathcal{O}_i) = R(\mathcal{O}_1) + R(\mathcal{O}_2)$

$\rightarrow \Delta(\mathcal{O}_i) \geq \Delta(\mathcal{O}_1) + \Delta(\mathcal{O}_2)$

dimensional analysis $\rightarrow C^i_{12}(x) \sim x^{d_i - d_1 - d_2}$

$\rightarrow$ no singularity at $x=0$

$\rightarrow$ take limit $x \rightarrow 0$: if non-vanishing we get

$\mathcal{O}_3$ with $\Delta(\mathcal{O}_3) = \Delta(\mathcal{O}_2) + \Delta(\mathcal{O}_1)$

$\rightarrow$ chiral operators form a ring!

R-symmetry will appear in the IR (non-anomalous) and commute with $SU(N_f) \times SU(N_f)$ flavor sym.

$\rightarrow$ for gauge invariant operators $Q\tilde{Q}$ and B we have:

$$\Delta(Q\tilde{Q}) = \frac{3}{2}R(Q\tilde{Q}) = 3\,\frac{N_f - N_c}{N_f}$$

$$\Delta(B) = \Delta(\tilde{B}) = 3\,\frac{N_c(N_f - N_c)}{2N_f}$$

From § 2.2 we know that chiral fields satisfy
$r \geq \frac{2}{3} + \frac{1}{3} j$ (consistency of L and $\bar{B}_1$ shortenings)
Hence $\Delta \geq 1$
saturated for free scalar field: $\Delta = 1$, $j = 0$
satisfying $\partial_\mu \partial^\mu \Phi = 0$
$\rightarrow$ $\Delta(\tilde{Q} Q) < 1$ for $N_f < \frac{3}{2} N_c$ $\rightarrow$ inconsistent!
For $N_f < \frac{3}{2} N_c$ the description of the physics changes
$\longrightarrow$ for $N_f = \frac{3}{2} N_c$ $\Delta(M = \tilde{Q} Q) = 1$
IR theory becomes free $\partial^2 M = 0$!

<u>The dual theory</u>:
SCFT has a dual UV description in term of another gauge theory for $N_c + 2 \leq N_f \leq \frac{3}{2} N_c$
We are searching for a gauge theory which becomes free free for $N_f \leq \frac{3}{2} N_c$ (see above)
$\rightarrow$ take gauge group $SU(N_f - N_c)$
$\rightarrow$ β-function becomes positive
For $N_f \geq N_c + 1$ $\mathcal{M}_{cl} = \mathcal{M}_{quantum}$
$\rightarrow$ vacuum at origin: $M = B = \bar{B} = 0$

with global symmetry

$$SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$$

$\rightarrow$ N_f quarks q transforming as $N_f - N_c$ rep.
and $\tilde{q}$ transforming as $\overline{N_f - N_c}$

From $B \sim q^{N_f - N_c}$ and $\tilde{B} \sim \tilde{q}^{N_f - N_c}$ we get

$$q \text{ in } \left(\overline{N_f}, 1, \frac{N_c}{N_f - N_c}, \frac{N_c}{N_f}\right)$$

$$\tilde{q} \text{ in } \left(1, N_f, -\frac{N_c}{N_f - N_c}, \frac{N_c}{N_f}\right)$$

(compare with table 1)

baryon number of q is fractional

$\rightarrow$ q is <u>not</u> polynomial of Q

R-charge assignment is anomaly free

Include meson fields as independent fields

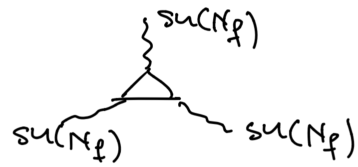
$$M \text{ in } \left(N_f, \overline{N_f}, 0, 2\left(\frac{N_f - N_c}{N_f}\right)\right)$$

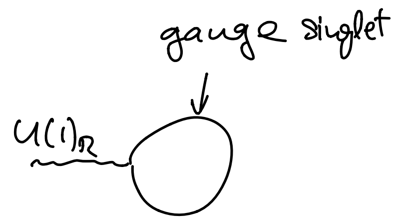
Superpotential:

$$W = M^i_{\tilde{i}}\, q_i\, \tilde{q}^{\tilde{i}}$$

$\rightarrow$ find both in the original $SU(N_c)$ and the dual $SU(N_f - N_c)$ gauge theory the same 't Hooft anomaly matching conditions!

$SU(N_f)^3$	$N_c\, d^{(3)}(N_f)$
$SU(N_f)^2 U(1)_R$	$-\dfrac{N_c^2}{N_f}\, d^{(2)}(N_f)$
$SU(N_f)^2 U(1)_B$	$N_c\, d^{(2)}(N_f)$
$U(1)_R$	$-N_c^2-1$
$U(1)_R^3$	$N_c^2-1-2\dfrac{N_c^4}{N_f^2}$
$U(1)_B^2\, U(1)_R$	$-2N_c^2$





For $\frac{3N_c}{2} < N_f < 3N_c$ $SU(N_f - N_c)$ gauge th. with N_f quarks is asymptotically free ($\beta < 0$)

$\longrightarrow$ flows to IR fixed point SCFT

$$\Delta(M) = 3\,\frac{N_f - N_c}{N_f}$$

As N_f becomes smaller original theory becomes strongly coupled, dual description becomes weakly coupled $\longrightarrow$ electric-magnetic duality

$q, \tilde{q}$ are non-Abelian magnetic monopoles of original theory

Check: "dual of dual is original theory"

start with $SU(N_c)$, N_f flavors

$$\downarrow \text{dualize}$$

$SU(N_f - N_c)$, N_f flavors, meson field M

$$\downarrow \text{dualize}$$

$SU(N_c)$, N_f flavors, mesan fields M, N

$$W = (M^i_{\tilde{i}} - Q^i \tilde{Q}_{\tilde{i}}) N^{\tilde{i}}_i$$

integrating out $N^{\tilde{i}}_i \longrightarrow M = Q\tilde{Q}$ definition of meson field

Summary:

for $3N_c/2 < N_f < 3N_c$ gauge theory flows to SCFT in the IR

two dual descriptions:

- electric is weakly coupled for $2N_c \leq N_f < 3N_c$
  free for $3N_c \leq N_f$
- magnetic is weakly coupled for $\frac{3}{2}N_c < N_f \leq 2N_c$
  free for $N_f \leq \frac{3}{2}N_c$